The role of wing geometry on batoid gait selection

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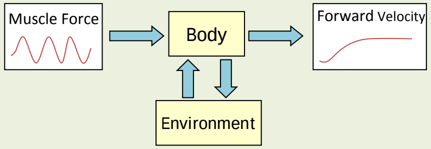
Abstract

A dynamic model of batoid swimming is developed and analyzed for three specific species: the Atlantic, Butterfly, and Cownose rays. These species are chosen for their close biological relationship and variety of observed swimming gaits, from undulation (passing waves down the fin) to oscillation (bird-like flapping). Each species is modeled using biological parameters, such as wing size and shape, approximate fluid forces, observed frequency of wing oscillation, and swim velocity. Optimal swimming gaits with respect to various criteria, including energy efficiency, muscle tension, and wing curvature, are calculated for each case and compared. A general trend is observed where undulatory gaits result for lower aspect ratio, elliptic wing shapes and oscillatory gaits result for higher aspect ratio, triangular wing shapes, which agrees with biology. Gait transitions are also observed as oscillation frequency is changed, from symmetric gaits with pitching of the body at higher frequencies to anti-symmetric gaits with rolling of the body at lower frequencies, both of which are observed in batoid swimming.

Batoid Model Overview

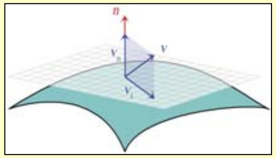
Framework: Mechanical Rectifiers


Muscle Force → Body → Forward Velocity
Body ⇅ Environment

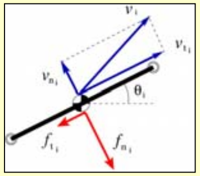
Periodic body motion + Environment Interaction = Velocity

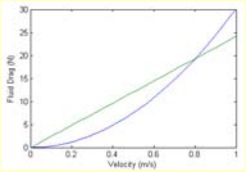
Environment: Normal and Tangential Fluid Forces

Surface Normal



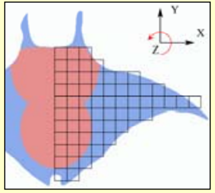
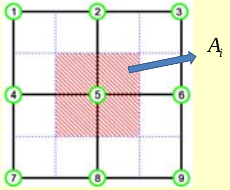
Linear Force Approximation $f_t = c_t v_t \qquad f_n = c_n v_n$





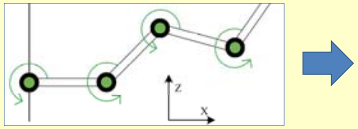
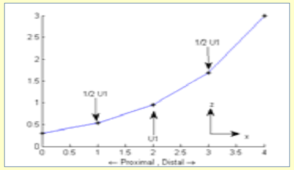
Body: Batoid Mechanical Model

Wing Discretization

Point masses placed at grid points with mass proportional to surrounding area.

Actuation: Torque Approximation

Small amplitude approximation: Point mass motion in body z direction only.

Equations of Motion

Nonlinear Equations of Motion

Wing Shape and Body Orientation

$$\underbrace{J(\theta)\ddot{\theta} + G(\theta,\dot{\theta})\dot{\theta}}_{\text{Inertial Terms}} + \underbrace{D(\dot{\theta}) + K(\theta)}_{\text{Wing Stiffness and Damping Terms}} + \underbrace{R(\theta)^T \gamma(R(\theta)\dot{\theta} + N(\theta)v)}_{\text{Environmental Fluid Forces}} = \underbrace{Bu}_{\text{Wing Muscle Actuation}},$$

Center of Mass Motion

$$\overset{\text{Total Mass}}{m_t}\overset{\text{C.M. Acceleration}}{\dot{v}} + \underbrace{N(\theta)^T \gamma(R(\theta)\dot{\theta} + N(\theta)v)}_{\text{Environmental Fluid Forces}} = 0, \quad \theta := \begin{bmatrix} z^b \\ \phi \end{bmatrix} \begin{matrix} \rightarrow \text{Point mass positions in body frame} \\ \rightarrow \text{Body orientation (Euler Angles)} \end{matrix}$$

Small Amplitude Approximation: Bilinear Equations of Motion

$$\left.\begin{matrix} J\ddot{\theta} + D\dot{\theta} + K\theta - L(\theta)v = Bu, \quad m_t\dot{v} + E\dot{\theta} + C(\theta)v = L(\theta)^T\dot{\theta} \end{matrix}\right\} L(\theta): \text{Linear in } \theta, \;\; C(\theta): \text{Quadratic in } \theta.$$

$$\theta \rightarrow \boxed{\int_0^T L(\theta)^T \dot{\theta}\, dt} \rightarrow \text{Thrust}$$

Optimal Gait Problem

Problem Statement

Given: Body, Environment, Desired Velocity
Find: Periodic Motion Minimizing a Cost Function

⇔

Reformulation

$$\min \frac{1}{T}\int_0^T \begin{bmatrix} \theta \\ u \end{bmatrix}^T \Pi \begin{bmatrix} \theta \\ u \end{bmatrix} dt \quad \text{s.t.} \quad \frac{1}{T}\int_0^T v\, dt = v_0$$

Quadratic Objective Function — Velocity Constraint

Quantity	Integral	Π
Deflection Rate	$\frac{1}{T}\int_0^T \lVert\dot{\varphi}\rVert^2\, dt$	$\begin{bmatrix} \omega^2 W^T W & 0 \\ 0 & 0 \end{bmatrix}$
Muscle Tension	$\frac{1}{T}\int_0^T \lVert u\rVert^2\, dt$	$\begin{bmatrix} 0 & 0 \\ 0 & I \end{bmatrix}$
Power	$\frac{1}{T}\int_0^T \dot{\theta}^T Bu\, dt$	$\frac{1}{2}\begin{bmatrix} 0 & -j\omega B \\ j\omega B^T & 0 \end{bmatrix}$

Solution: Generalized Eigenvalue and Eigenvector

$$u(t) := \Re\left(q_0 e^{j\omega_0 t}\right), \qquad q_0 := \sqrt{\xi_0^* Y \xi_0}^{\,-1}\xi_0, \qquad \left(X_{\omega_0} - \gamma_0 Y_{\omega_0}\right)\xi_0 = 0, \qquad \gamma_0 := \min_{\omega\in\Re}\max_{\lambda\in\Re}\{\lambda : X_\omega \geq \lambda Y_\omega\}$$

Optimal Gait Studies

Model Parameters

Biological Measurements[1]	Atlantic (Undulator)	Butterfly (Mixed)	Cownose (Oscillator)
Disc Width	.2 m	.26 m	.32
Disk Length	.19 m	.13 m	.15
Tip Amplitude Factor	.15	.2	.3
Oscillation Frequency	15.7 rad/s	8.38	7.85 rad/s
Desired Speed: 2 DL/s	.38 m/s	.26 m/s	.3 m/s

Optimal Gait Data

Number of Waves

Undulation

	Energy Efficiency	Deflection Rate	Muscle Tension	Biology
Atlantic	1.0231	0.69658	0.51095	1.3
Butterfly	0.46485	0.30668	0.23985	.6
Cownose	0.30962	0.24847	0.24128	.4

Oscillation

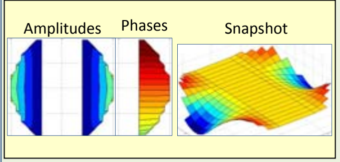
Wing Tip Amplitude

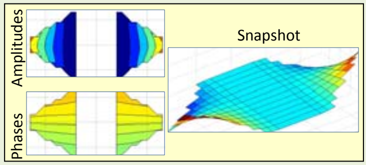
Undulation

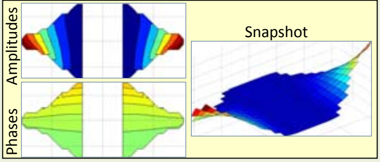
Energy Efficiency	Deflection Rate	Muscle Tension	Biology
0.034007	0.04598	0.055237	.03
0.039563	0.043826	0.050368	.05
0.04419	0.051735	0.063619	.09

Oscillation

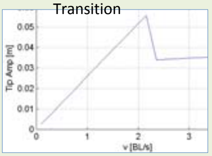
Atlantic ← **Undulation** **Butterfly** **Oscillation** → **Cownose**



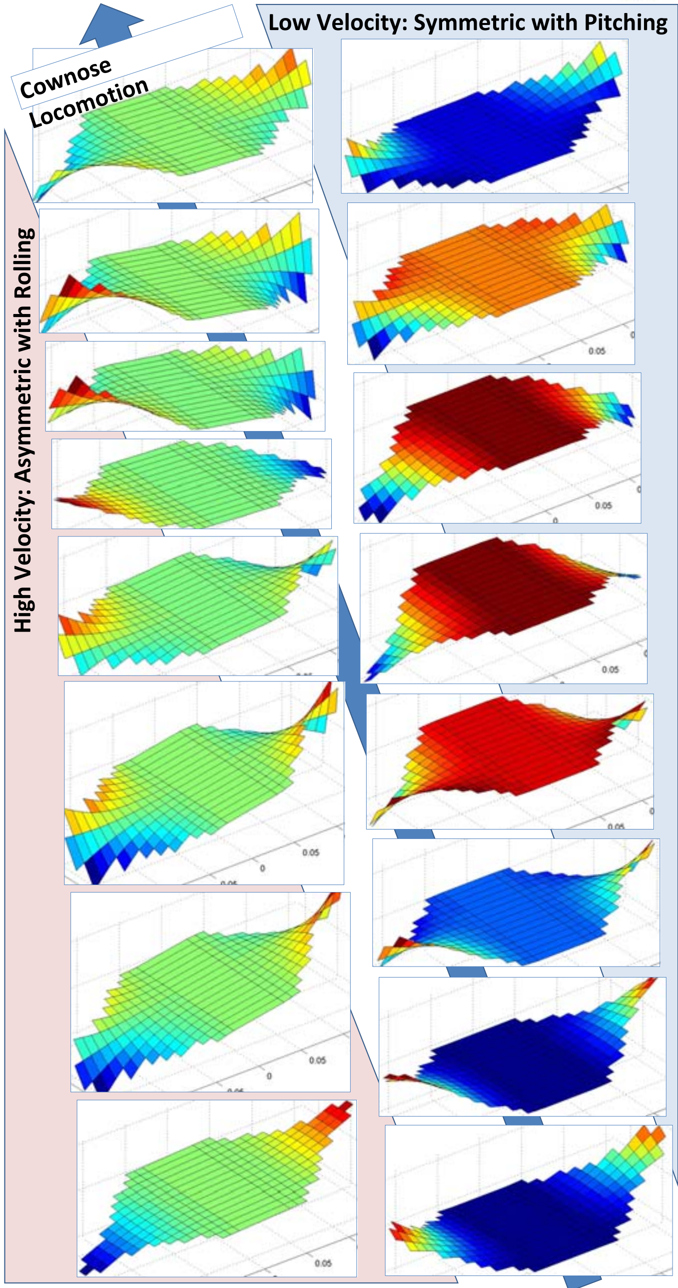




Cownose: Amplitude Drop Across Symmetric - Asymmetric Transition



Cownose Locomotion



Conclusion

We have shown lower aspect ratio, elliptic wing shapes result in more undulatory gaits for all cost functions (and the converse). Similarly, the cost functions also influence the resulting gait, with energy efficiency being the most undulatory, muscle tension being the most oscillatory, and wing curvature in the middle. Gait transitions were also observed as the locomotion velocity to frequency ratio was changed, from symmetric gaits with pitching of the body at lower velocities (or higher frequencies) to anti-symmetric gaits with rolling of the body at higher velocities (or lower frequencies), with an accompanying drop in amplitude. This is observed in batoid swimming during sudden acceleration, when the maximum thrust with minimal motion is required.

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[1] Rosenberger, L. J. (2001). *Pectoral Fin Locomotion In Batoid Fishes: Undulation Versus Oscillation.* The Journal of Experimental Biology, **204(2), 379–394.**